Phys 410
Spring 2013
Lecture #9 Summary
11 February, 2013

We considered energy for motion in one-dimensional systems. This is not as artificial as it first appears – later we will be able to break certain 3D problems to simpler 2D and 1D problems, and the methods that follow will be very useful. Consider a particle confined to move only on the x-axis. It has a kinetic energy $T = \frac{1}{2} m\dot{x}^2$. The kinetic energy can be altered by applying a force and doing work on the particle. The tangential component of the force does work as $W(x_0 \to x_1) = \int_{x_0}^{x_1} F_{\tan}(x')\,dx'$. If this force is conservative, one can define an associated potential energy (PE) as $U(x) = -W(x_0 \to x) = -\int_{x_0}^{x} F_{\tan}(x')\,dx'$, where it is assumed that $U(x_0) = 0$. We also expect that the total mechanical energy will be conserved: $E = T + U(x)$, and $\Delta E = 0$. This conservation law allows elegant solution of 1D problems involving conservative forces.

We did an example of a Hooke's law restoring force in 1D: $\vec{F} = -k\,x\,\hat{x}$., with an equilibrium point at $x = 0$. The corresponding potential is $U(x) = \frac{1}{2} kx^2$, with $U(0) = 0$. The mechanical energy is conserved: $E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2$. As the particle moves it exchanges energy back and forth between kinetic and potential energies.

Note that the force can be derived from the potential energy function through the 1D gradient, which is a total derivative: $F_{\tan} = -dU(x)/dx$.

The energy landscape created by the function $U(x)$ is very revealing. If there is a maximum or minimum in $U(x)$ it means that the driving force at that location is $F_{\tan} = 0$. As such, this represents an equilibrium point. A minimum in $U(x)$ is a stable equilibrium because a small displacement will result in forces that point back to the equilibrium point. This is the case when $d^2U(x)/dx^2 > 0$. A local maximum in $U(x)$ is unstable because a small displacement in either direction produces forces that draw the particle further away. This is the case with $d^2U(x)/dx^2 < 0$.

So far we have no mention of time in the evolution. We can find $x(t)$ starting with the mechanical energy. We showed that the statement of mechanical energy conservation can be rewritten as $t=\sqrt{\frac{m}{2}}\int_{x_0}^{x}\frac{dx'}{\sqrt{E-U(x')}}$. This can be solved for $x(t)$, and from that one can determine the velocity, acceleration, etc. as functions of time. We did problem 4.28 from HW 3 and solved the above equation for $x(t)$ for a simple harmonic oscillator.